MATHEMATICS D

Paper 4024/11
Paper 11

Key messages

In order to do well on this paper, candidates need to

● have covered the whole syllabus
● have accurately learned the necessary formulae
● be able to perform calculations accurately
● clearly show the working for each question
● give answers in the form required by the question.

General comments

There were many well-presented scripts of a good standard. Although some candidates did not attempt some of the questions towards the end of the paper most candidates appeared to have sufficient time to complete the paper.

Some questions were accessible to all candidates while others provided a challenge to more able candidates.

Having covered all the syllabus, candidates should feel confident to be able to make some attempt at all the questions and they should be encouraged to do so even when the question seems unfamiliar.

Candidates at all levels had difficulty with some aspects of the question involving Venn diagrams.

Comments on specific questions

Question 1

(a) This part was usually well done although some candidates were unable to place the decimal point correctly.

(b) Most candidates answered this correctly although a few inserted more than one pair of brackets and so lost the mark.

Answers: **(a)** (0).016 **(b)** $2 \times (3 + 4) \times 5$

Question 2

(a) This part on finding the perimeter of a shape proved challenging to many. Successful candidates adopted a systematic approach going round the perimeter of the shape and noting down the lengths, realising that the lower indentation made no difference to the perimeter and that the upper only added 2 cm. Sometimes the correct lengths were noted and then added incorrectly. 20 was a common incorrect answer from 6 + 4 + 6 + 4 and 24 from 6×4 was also seen.

(b) Some candidates forgot what a trapezium looks alike and several other shapes were drawn including parallelograms and rectangles. It was common to see a trapezium whose upper edge was 4 cm and some candidates seemed confused about the area of a trapezium.

Answers: **(a)** 22 **(b)** Any trapezium of area 18 cm² with height 4 cm and other parallel side 3 cm long

Question 3

About half of the candidates were awarded 2 marks on this question on angles. Many others earned 1 mark for stating or showing that $B\hat{D}C$ = 25° or $D\hat{A}E$ or $D\hat{E}A$ = 45°. Some went on to assume incorrectly that Δ DEC was isosceles and gave 25° as their answer and basic arithmetic errors were seen e.g. 180 – 45 = 125.

Answer: 20

Question 4

(a) Most candidates answered this question on prime factors correctly.

(b) Most candidates answered this correctly. A few confused sum with product and gave 3 and 5 as their answer.

Answers: **(a)** $2 \times 2 \times 3 \times 3$ **(b)** 2 and 13

Question 5

Many candidates were very successful with this question giving an equation as requested and solving it correctly. A few ignored the instruction to write down an equation but obtained the correct value of t and hence they lost a mark. Some candidates struggled with converting 2 hours 20 minutes into minutes, an integral part of this question. Some basic arithmetic errors were seen e.g. 140 ÷ 4 = 25, 35.5 etc.

Answer: t + 3t = 140 or 4t = 140 leading to (t =) 35

Question 6

This question was answered very well by those candidates who knew the facts about the different quadrilaterals. Many were very confused and gave incorrect answers. Some successful candidates drew a few sketches of possible quadrilaterals and this helped them to arrive at the correct answers.

Answers: **(a)** kite **(b)** parallelogram

Question 7

Many candidates answered this question correctly and found it helpful to extend the line *AB*. A lot of candidates calculated 66° and 37°, for which some credit was given, but then went on to add them together and subtract from 360° giving the incorrect answer of 257°. Others, having calculated 257° subtracted that from 360° to get 103° as their answer. Some correct methods were spoiled by arithmetic errors.

Answer: 77°

Question 8

(a) Most candidates attempted this part but many incorrect responses were seen. A lot of incorrect answers resulted from candidates calculating 84 × 60 and then attempting to convert this figure into metres giving answers involving the digits 504 e.g. 5.04, 5 040, 504 000. A few candidates, having correctly calculated 84 ÷ 60, then made an error in the conversion into metres resulting in answers e.g. 1.4, 14 and 140.

(b)   Many candidates answered this question correctly. Others still have problems in understanding the concept of finding the lower bound given a particular degree of accuracy and need to improve in this area. Candidates need to appreciate the significance of the degree of accuracy given in a particular question.

Answers: **(a)** 1400 **(b)** 12.25

Question 9

(a)   The majority of candidates obtained the correct answer in this probability question.

(b)   In this question candidates need to remember that the number of blue pegs is unchanged and this provides the easiest approach. 24 ÷ 0.2 gives the total number of pegs and thence the number of red pegs which need to be added. Some candidates attempted an algebraic method but forgot that if x pegs are added the total then is $40 + x$.

Answers: **(a)** 16 **(b)** 80

Question 10

(a) (i)   The majority of candidates gave the correct answer in this question on rounding.

(ii)   While many candidates were able to perform the required approximation others need to do more work on this topic. Candidates need to appreciate the difference between 250, which is correct, and 250.00, which is incorrect. 25 was another incorrect answer sometimes seen.

(b)   The majority of candidates were successful in obtaining the correct answer. Wrong answers came from calculating 8.36 + 63.58 and then dividing the result by 3 or from attempts to approximate the cube root of this sum. There seemed to be some confusion over the meaning of $\sqrt[3]{}$.

Answers: **(a)** 248.37 **(b)** 250 **(c)** 6

Question 11

This question on simultaneous equations was answered well by many candidates with correct solutions. Both elimination and substitution methods were seen. In the former the coefficients of x or y were equated correctly but errors occurred when subtracting or adding to eliminate x or y. Sign errors sometimes occurred when candidates chose to substitute for y from the second equation into the first equation.

Answer: $x = -2$, $y = 3$

Question 12

(a) (i)   Candidates need to improve their understanding of frequency density. Some candidates showed a appreciation of the topic by realising that the area of the first bar showed the number of people at the party aged b years where $5 < b \leqslant 10$ and worked out 5×1.0 to get $p = 5$. Others who gave the answer $p = 10$ took no account of the width of the bar.

(ii)   As with **(i)** candidates need to appreciate that the width of the bar is important in finding its area. Many ignored this and simply worked out 10×0.8

(b)   Candidates were more successful with this part and the majority completed the histogram correctly. A few misread the scale and draw a bar of height 1.2 rather than 1.4. Candidates seem to be more able to draw a histogram from given data rather than to interpret the information shown in one.

Answers: **(a)(i)** 5 **(ii)** 16 **(b)** Histogram completed correctly with column of height 1.4 for $20 < b \leqslant 30$

Question 13

(a)   Most candidates answered this question on fractions correctly. A few made basic arithmetic errors e.g. 24 – 5 = 17

(b)   Most candidates answered this correctly. Some worked out $\dfrac{2}{5} \times \dfrac{3}{7}$ which was not the correct calculation.

(c)   This part proved more challenging. A popular method was to add the two fractions to obtain $\dfrac{31}{24}$ and then divide the sum by 2 resulting sometimes in the incorrect answer $\dfrac{31}{12}$ but most obtained the correct answer. A few chose to subtract or even multiply the two fractions together. $\dfrac{15\frac{1}{2}}{24}$ was not an acceptable answer and neither was $\dfrac{15.5}{24}$ as we need integers in a fraction answer.

Answers: **(a)** $\dfrac{19}{40}$ **(b)** $\dfrac{14}{15}$ **(c)** $\dfrac{31}{48}$

Question 14

(a)   Most candidates answered this question on standard form correctly although 1.86×10^4, 1.86×10^3, and 186 were all seen.

(b)(i)   Many candidates answered this correctly. They need to remember that the question asks for the answer in standard form and also basic arithmetic skills such as $8^2 = 64$ and not 16.

(ii)   The placing of the decimal point was crucial in this question. The candidates needed to realise that $8 \times 10^8 = 80 \times 10^7$ and so the required calculation was $80 - 1.3 = 78.7$ and not $8 - 1.3$ which gave the incorrect digits 6.7. Incorrect answers involving the digits 67 were common.

Answer: **(a)** 1.86×10^{-4} **(b)(i)** 6.4×10^{17} **(ii)** 7.87×10^8

Question 15

(a)   This part was generally well done with candidates using the given formula for the volume of a pyramid. Some errors occurred after $\dfrac{1}{3}(12 \times 9) \times 30$ because candidates did not cancel the 3 into the 30 and after 3240/3 obtained the wrong answer of 108

(b)(i)   Many candidates ignored the request for a fraction here and answers such as 10 (from 30 ÷ 3) and 360 (from 1080 ÷ 3) were often seen. Candidates need to understand the relationship between the volumes of similar solids. This was not fully understood as the answers $\dfrac{1}{3}$ and $\dfrac{1}{9}$ were seen.

(ii)   The expectation here was that candidates would use their fraction from **(i)** to find the volume of M and then subtract that from the volume of L to find the volume of N. Hence **(i)** is intended to be an aid to answering **(ii)**. Some candidates adopted this approach and those working with the correct fraction from **(i)** often got the correct answer. Other candidates started again and found the volume of a pyramid with height 20 cm and mistakenly thought that this would give them the correct volume of N.

Answers: **(a)** 1080 **(b) (i)** $\dfrac{1}{27}$ **(ii)** 1040

Question 16

(a) (i) This part on substituting values was usually correct.

(ii) This part was usually correct too. There were a few errors in calculating $(-7)^2$ => 42 or –49 or 14.

(b) The majority of candidates were able to rearrange the formula correctly. They should take care when writing their answer to ensure that the square root sign encompasses the whole fraction. Some candidates made a sign error and got $2r^2 = 5 - A$ and others were not sure how to obtain r from r^2 and divided by 2 instead of square rooting.

Answers: **(a)(i)** 13 **(ii)** 58 **(b)** $(\pm) \sqrt{\dfrac{A-5}{2}}$

Question 17

(a) In order to succeed in this part candidates need to know which line is given by $y = -1$. More work needs to be done on this topic as candidates did not know this and reflections were seen in various lines including the y axis, the x axis, $y = -\dfrac{1}{2}$, $x = -1$ and $x = -\dfrac{1}{2}$. Rotations and translations were also seen.

(b) (i) This was a challenging part and was omitted by many. Candidates needed to realise that points move parallel to the x-axis with a stretch when the y-axis is invariant and this did not seem to be fully understood.

(ii) Another challenging part which many candidates were not able to attempt.

Answers: **(a)** B drawn with vertices (2, –3)(3, –3)(3, –5) **(b)(i)** C drawn with vertices (4, 1)(6, 1)(6, 3)

(ii) $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$

Question 18

(a) (i) This part on evaluating indices was usually correct. A few candidates showed $\dfrac{1}{9}$ in their working and 9 on the answer line as if they were not sure which to give as their answer. A few also tried to work in decimals giving the answer 0.1 which was not acceptable. Candidates should realise that it is best practice to answer such questions with a fraction answer.

(ii) This part was usually correct. A few candidates seemed confused by the notation and worked out $125 \times 2/3$.

(b) Many candidates simplified this fraction correctly and others made a good start by obtaining $\dfrac{b^4}{9a^2}$ but then forgot to square root. A systematic approach to the problem is needed, dealing in turn with the numbers and then the two letters and remembering to raise it all to the power $\dfrac{1}{2}$ at the end. Some candidates need more practice at this type of problem.

Answers: **(a)(i)** $\dfrac{1}{9}$ **(ii)** 25 **(b)** $\dfrac{b^2}{3a}$

Question 19

(a) This part on units and area was usually answered correctly. Some answers were spoiled by basic arithmetic errors and a few did not read the question carefully and though that $258 was the cost of 1 square metre of carpet.

(b) Many correct answers were seen. Some candidates multiplied 90 by 0.3 while others realised that they needed to divide 90 by 0.3 but got the answer 30.

(c) This part proved challenging for many. Many candidates thought that the time and the rate were in direct proportion and did not understand that a higher flow rate would lead to a shorter time. The majority of candidates calculated $\dfrac{4 \times 20}{2.4}$ giving the answer $33\dfrac{1}{3}$ which was not a sensible answer within the context of the question. Candidates should be encouraged to always think about whether or not their answer is sensible.

Answers: **(a)** 430 **(b)** 300 **(c)** 12

Question 20

(a) Some candidates found this question on travel graphs difficult and arithmetic errors were common. About half obtained the correct answer.

(b) Most candidates realised that the distance travelled was 25 km and that the time for the journey was 50 minutes but had difficulty processing this information. The answer 0.5 was frequently seen from 25/50. Some candidates misread the graph and thought that the time taken was 55 mins or $\dfrac{3}{4}$ hour or 11 hrs 55 mins. Several candidates omitted this part.

(c) (i) Only the most able candidates drew the correct line. The common error was to draw a line going from (1125, 0) to (1155, 25) misinterpreting the fact that the yellow bus leaves from the town.

(ii) Again few earned marks here as the wrong line had been drawn in **(i)**.

Answers: **(a)** 11 **(b)** 30 **(c)(i)** line joining (1125, 25) to (1155, 0) **(ii)** 1136 – 1137

Question 21

(a) (i) Many candidates showed a lack of understanding of how to represent information in a Venn diagram. Some correct diagrams were seen but the majority of candidates did not take an overall view of the situation and began with putting 24 into $F \cap (B \cup C)'$, 28 into $C \cap (B \cup F)'$ and 16 into $B \cap (C \cup F)'$. They then went on to put 9 into $(B \cap C) \cap F'$, 11 into $(C \cap F) \cap B'$ and 6 into $(B \cap F) \cap C'$ finishing up with 5 in $F \cap B \cap C$ and 8 in the correct area. Candidates had not checked their completed Venn diagram to see if it correctly represented the information given.

(ii) Many candidates who had the correct Venn diagram in **(a)(i)** got the correct number of members. Wrong answers were common, usually 99 or 107, from incorrect Venn diagrams.

(b) (i) Candidates did slightly better in this part, when they were given the completed Venn diagram, and obtained the correct answer. Some forgot to include those not playing baseball, cricket or football and gave the answer 36

(ii) Many candidates did not know which sections of the Venn diagram to include and many different incorrect answers were seen with no indication of where they had come from. A few candidates included those playing none of the games and got the answer 43.

Answers: **(a)(i)** Correct Venn diagram **(ii)** 55 **(b)(i)** 40 **(ii)** 39

Question 22

(a) This part on the mid-point of a line was usually correct.

(b) Some candidates omitted this part. Candidates did not seem to have a thorough understanding of translation as they had difficulty applying it to the line AB. Very few used the diagram which was given at the start of the question, to help them in this part and in **(c)**.

(c) Again some candidates omitted this part and few found the correct co-ordinates of *G.* More work is needed on rotation where the centre of rotation is not (0, 0)

(d) More candidates were successful in this part and many earned at least 1 mark for getting $y = \frac{1}{3}x + k$ if not the completely correct equation.

Answers: **(a)** $\left(3\frac{1}{2}, 3\frac{1}{2}\right)$ **(b)** (−1, 4) **(c)** (1, 0) **(d)** $y = \frac{1}{3}x + \frac{7}{3}$

Question 23

Vectors continue to be a topic which many candidates find difficult and more work needs to be done so that they are able to attempt questions with greater confidence.

(a) (i) Many candidates were able to answer this part correctly. A few gave answers such as **p** + **q** and **p** − **q**.

(ii) Some candidates omitted this part. The question stated that PS : SR = 1 : 3 and many candidates interpreted this incorrectly as PS = $\frac{1}{3}$ PR and went on to get **p** − $\frac{2}{3}$ **q** as their answer.

(b) (i) Many candidates omitted this part. Candidates should be encouraged to begin by finding a route which takes them from *P* to *T* via *O*, *P*, *Q* and *R*. Then they can try to find the linking vectors. Some candidates earned marks from this approach even when they did not manage to obtain the correct final answer.

(ii) Some of the candidates who answered **(b)(i)** correctly were able to draw the correct conclusion but many omitted this part as well as **(b)(i)**.

Answers: **(a)(i)** **q** − **p** **(a)(ii)** **p** − $\frac{3}{4}$ **q** **(b)(i)** $\overrightarrow{PT} = \frac{1}{3}$ **p** **(ii)** *O, P* and *T* are collinear

Question 24

(a) This part was generally well done with many candidates solving the equation correctly. A few forgot to multiply the whole 'bracket' by 5 and got 5*x* + 1 etc.

(b) (i) This part was well answered.

(ii) There were some excellent answers to this part. Some candidates got as far as $(m − 3)^2 + 1 = 17$ but then thought that $(m − 3)^2$ was equal to $(m + 3)(m − 3)$. Most candidates, having got to $m^2 − 6m − 7 = 0$ were able to factorise correctly and obtain the correct solutions to the equation.

Answers: **(a)** 23 **(b)(i)** −8 **(ii)** −1 or 7

MATHEMATICS D

Paper 4024/12
Paper 12

Key messages

In order to do well in this paper, candidates need to

- be familiar with the entire syllabus and remember necessary formulae
- be competent at basic arithmetic
- write their answers clearly on the answer line
- be able to demonstrate good geometrical reasoning.

General comments

Many candidates demonstrated good understanding of most of the topics covered in the paper. Candidates had time to complete all of the questions and many produced well-presented responses. The paper discriminated between candidates of different abilities. The more able candidates were able to access the most challenging questions but there were many questions that were accessible to all candidates.

Candidates demonstrated a high level of skills in algebra. They found questions involving geometric reasoning more challenging. Questions that required a problem solving approach were found to be difficult by many and some work in these questions was set out in a confused manner. Areas of particular weakness were upper and lower bounds, set notation, angles in polygons and vectors.

Some candidates were let down by errors in basic arithmetic. Errors such as $20 \times 30 = 60$ and $720 \div 120 = 7$ were not uncommon. In questions involving algebra, errors in rearrangement of negative terms were also common.

Some candidates continue to write their work in pencil and then overwrite this working in pen. This often leads to work that is not legible and should be avoided. Work is clearer if candidates write everything in pen and cross it out and replace it if an error is made. Pencil should only be used for graphs and diagrams. Some candidates' figures were difficult to read: in some cases it was difficult to distinguish between 1, 4 and 7 or 5 and 8.

Comments on specific questions

Question 1

(a) Most candidates subtracted the fractions correctly. Most candidates used a common denominator correctly and incorrect answers usually resulted from arithmetic errors in the subtraction of the numerators. Some tried to convert $\frac{7}{15}$ to a decimal which was not required. A small number of candidates simply subtracted the given numerators and denominators leading to an answer of $\frac{3}{2}$.

(b) Most candidates answered correctly, although some answers included unnecessary zeros, for example 000.0012. Some answers were given in fraction form which was accepted. Most decimal answers involved the figures 12 and the errors seen were in the place value.

Answers: **(a)** $\frac{7}{15}$ **(b)** 0.0012

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Question 2

(a)   Many candidates were unable to use line symmetry to complete the pattern correctly.

(b)   Candidates found rotational symmetry more challenging than line symmetry and many incorrect responses were seen.

Answers: **(a)** correct triangle shaded **(b)** two correct triangles shaded

Question 3

Many candidates showed the three numbers written correct to one significant figure and carried out the calculation to reach the correct answer. Some candidates were unable to position the decimal point correctly in the final answer, and answers of 0.3 or 0.003 were common. The most common incorrect approximations were to use 59 in place of 60 or 4.1 in place of 4. A number of candidates did not understand what was required in an estimation and tried to carry out long multiplication and division on the given numbers.

Answer: 60, 4 and 20 used leading to 0.03

Question 4

Many candidates identified that the angle in the pie chart for yes was 280° and used the unitary method with the given values to calculate the number of students who said yes. Some candidates found the total number of students represented in the pie chart. Errors were often the result of incorrect use of 360 in the division.

Answer: 700

Question 5

(a)   Candidates who had a protractor usually measured the angle accurately. Some candidates read the wrong scale on the protractor or gave an answer in centimetres.

(b)   Although village C was not marked on the diagram, candidates could have used the given diagram to help find the required bearing. Few candidates did this and an incorrect answer of 095° resulting from 360 – 265 was common. The correct subtraction of 180 from 265 was rarely seen.

Answers: **(a)** 137° **(b)** 085°

Question 6

(a)   Many candidates have difficulty in understanding upper and lower bounds and the inclusion of negative numbers in this question made it more challenging. There were a significant number of correct answers but the lower bound of –8.5° was often confused with the upper bound.

(b)   Some candidates appreciated that the lower bound of the difference resulted from subtracting the highest outside temperature from the lowest inside temperature. The most common misconception was to subtract –8 from 10 leading to 18 and either giving this as the answer or subtracting 0.5 from 18 to give 17.5 as the answer.

Answers: **(a)** –7.5 °C **(b)** 17 °C

Question 7

(a)   Although many candidates gave the correct response, the incorrect answer of $A \cup B'$ was almost as common. There were some cases where candidates were not sure of what symbol to use for 'not' or where to position it.

(b) Candidates found this part challenging, with few identifying that *D* was a subset of *C*. Some wrote the word subset in the answer space but were unable to write the correct symbol which was required. Common incorrect symbols seen were ∈, ∪, < or €.

Answers: **(a)** $A \cap B'$ **(b)** ⊂

Question 8

(a) Many candidates could work out the time difference correctly. Common incorrect answers were 3 h 45 m, 3 h 15 m and 2 h 55 min. A small number of candidates subtracted the times as if they were decimals and gave the answer 21 h 15 m.

(b) Although many candidates identified the calculation required in this part and divided to get 16.6…, many found it difficult to identify which date should be the final answer. As well as the correct answer of 17 May being frequently seen, answers of 16 May and 18 May were also very common. It was common to see the multiplication of 20 and 30 incorrectly evaluated as 60 rather than 600. Some candidates inverted the division to 600 ÷ 10 000. A number of candidates attempted to use a day-by-day approach to the calculation but this was rarely successful.

Answers: **(a)** 2 hours 45 minutes **(b)** 17 May

Question 9

(a) Many candidates gave the correct answer. Common errors were to include –1.5 or 2 or to omit 0. Some candidates clearly did not understand the term 'integer' and gave a list of decimals, such as –1.5, –1.4, –1.3…, as their answer.

(b) Many correct answers were seen in this part, with answers such as $\frac{2}{3}$, $\frac{3}{5}$, $\frac{5}{8}$, $\frac{6}{10}$ and $\frac{13}{20}$ common. Incorrect answers such as $\frac{1}{4}$, $\frac{1}{3}$, $\frac{3}{2}$, $\frac{5}{4}$, decimals such as 0.6 or even just *x* were also seen.

(c) Many candidates do not know what an irrational number is. Correct answers were rare but were usually $\sqrt{5}$ or $\sqrt{6}$. The most common answers in this part were 2.5 or $\frac{5}{2}$, where candidates were clearly confused between irrational numbers and improper fractions. Some candidates gave recurring decimals as answers. Irrational numbers that were outside the given range, such as π or $\sqrt{2}$ were also sometimes seen.

Answers: **(a)** –1, 0, 1 **(b)** fraction between $\frac{1}{2}$ and $\frac{3}{4}$ **(c)** irrational number between 2 and 3

Question 10

(a) Many candidates found the median correctly. The most common error was to find the value halfway between 192 and 202, the middle values in the list, rather than first ordering the given numbers. Some candidates ordered the list but thought that 185 was the middle number and others added 185 and 189 but did not divide by 2. A small number of candidates found the mean rather than the median.

(b) Candidates who understood that they could use the mean mass to calculate the total mass of carrots in the bag usually reached the correct answer. Candidates attempting the correct method sometimes made arithmetic slips in the calculation of 60 × 5 or 65 × 6, but some were confused about which number to multiply by 5 and which by 6. Common incorrect methods led to the answer 5, from 65 – 60, or 10.8, from 65 ÷ 6.

Answers: **(a)** 187 g **(b)** 90 g

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Question 11

Many candidates were able to solve the simultaneous equations correctly and showed clear, correct algebra. Those eliminating *y* were more successful than those eliminating *x*. Those who solved the simultaneous equations by substitution sometimes made errors when multiplying out the bracket or when eliminating the fraction. Candidates making errors in reaching their first variable were sometimes successful in following through to their second variable. Some candidates made errors when dealing with negative signs.

Answer: $x = 3$, $y = -\dfrac{1}{2}$

Question 12

(a) Many candidates identified that the equation would be in the form $y = \dfrac{k}{x^2}$ and often chose a correct pair of values from the table to find the value of *k*. Not all candidates substituted this value of *k* into their final answer and gave the final answer as $y = \dfrac{k}{x^2}$. Some candidates tried to use the values in the table to find the equation of a line in the form $y = mx + c$.

(b) The correct answer was often seen. Some candidates left their answer as $\sqrt{\dfrac{1}{4}}$, which was not acceptable. Some candidates who had not known how to answer part **(a)** used the values in the table to answer this part correctly.

Answers: **(a)** $y = \dfrac{12}{x^2}$ **(b)** $\dfrac{1}{2}$

Question 13

(a) Almost all candidates answered this part correctly.

(b) Although many candidates quoted a correct formula for calculating simple interest, many were unable to substitute the correct values into the formula in this reverse problem. Many used the total of 162, rather than the interest of 162 – 150, in the formula which led to the incorrect answer of 27. Others substituted correctly, but made errors in their rearrangement. A very common incorrect answer was 3 which is the amount of interest per year rather than the percentage rate of interest.

Answers: **(a)** $150 **(b)** 2%

Question 14

(a) Many candidates were able to substitute the numbers correctly into the function and rearrange to find the correct value of *k*. The most common error was to substitute 7 for *x* and 11 for f(*x*) and a number of candidates did not substitute any value for f(*x*). Arithmetic errors when rearranging to find the result were not uncommon.

(b) Most candidates understood how to find the inverse function and correct answers were common. Some answers were given in terms of *y* rather than *x*, which is not acceptable for full credit.

Answers: **(a)** 5 **(b)** $\dfrac{4x + 5}{3}$

Question 15

(a) Many candidates identified the transformation as a reflection and some gave the correct equation of the mirror line. Some incorrect equations such as *y* = *x* or *y*-axis were given. Some candidates added centre (0, 0) which is not acceptable as part of a correct description of the reflection. It was

common to see the transformation incorrectly described as a rotation of 180°. Very few candidates gave two transformations in their description.

(b) Many candidates carried out the correct rotation in this part. Some candidates carried out a 90° anti-clockwise rotation about an incorrect centre and others carried out a 90° clockwise rotation about the origin, both of which were given partial credit.

Answers: **(a)** reflection in *y* = –*x* **(b)** correct rotation

Question 16

(a) Many candidates were able to select the appropriate coordinates to find the equation of the line correctly. Those who did not give a correct equation usually found the gradient correctly. A small number of candidates attempted to find the length of the line.

(b) Many candidates were able to use the coordinates given for points *B* and *C* and the given gradient to set up an equation for *p* in the form $\frac{-1-5}{p-1} = -\frac{3}{4}$ which was sufficient to gain the method mark. However errors in rearrangement of this equation were common and many candidates did not reach the correct answer. Some candidates set up an equation using the coordinates of point *B* in the form $5 = -\frac{3}{4} \times 1 + c$ to find the value of *c* and then went on to set up a second equation with the coordinates of point *C* to find the value of *p*: this method often contained errors and rarely led to the correct answer. A small number of candidates used (change in *x*) ÷ (change in *y*) as the gradient.

Answers: **(a)** *y* = 2*x* + 3 **(b)** *p* = 9

Question 17

(a) In questions asking for a geometrical reason, candidates are expected to give a precise reason using correct terminology. In this case, the syllabus states 'angles in the same segment are equal' and this is what was expected. The answer 'angles subtended by the same arc' was also accepted. Reasons such as 'angles in a segment' or 'angles in a sector' or 'angles on the same chord' were not accepted. Expressions such as 'butterfly property' will never be accepted as a geometrical reason. Some vague responses such as 'angles coming from the same point' or 'angles on the same circumference' were also seen.

(b) Many candidates used the property of angles in a semicircle to find this angle correctly. A reason was sometimes given, but was not required in this part.

(c) Again, many candidates were able to find this angle correctly, with common wrong answers of 55 and 62.5 seen.

(d) Many candidates used the properties of angles in opposite segments to find this angle correctly. Some gave the answer 90° because they thought it was an angle in a semicircle.

Answers: **(a)** angles in same segment are equal **(b)** 55° **(c)** 70° **(d)** 110°

Question 18

(a) Most candidates understood that they needed to set up an expression for the gradient of the line to find the value of *v* and many reached the correct answer. The most common error was to use $\frac{12-v}{15}$ for the gradient leading to the answer *v* = 6. Some candidates used 25 or 10 rather than 15 for their denominator.

(b)   Most candidates understood that distance is given by the area under the graph and many reached the correct answer. Those who didn't reach the correct answer usually showed enough working to be given credit for a partial area found correctly, usually one of the rectangles of area 180, 120 or 300. Only a very small number of candidates attempted an incorrect calculation, usually a division from an incorrect rearrangement of speed = distance ÷ time.

Answers: **(a)** *v* = 18 **(b)** 345 m

Question 19

(a)   Many candidates set up an equation with 9*x* as the sum of the given angles. Many were not able to find the sum of the angles in a pentagon as 540°. Common errors were the incorrect calculation of 180 × 3 or use of an incorrect formula such as 180(*n* – 1). Some candidates calculated 540 correctly, but then divided it by 3 or 5 before equating with 9*x*. It was also common to see 9*x* equated with 360°, perhaps because candidates thought the sum of angles in any polygon is 360°, which led to the answer 40°.

(b)   Candidates found it difficult to interpret the information presented in this question and many did not reach the correct answer. A common error was to take 30° as the exterior angle of the polygon and calculate 360 ÷ 30 = 12 as the number of sides. Another common error was to take 150° as the interior angle of the polygon. Those candidates who used the symmetry of the diagram to calculate the missing angles of 165° and indicate them on the diagram often went on to set up the equation $165 = \frac{180(n-2)}{n}$ which they were able to solve to give *n* = 24.

Answers: **(a)** *x* = 60 **(b)** *n* = 24

Question 20

(a)(i)   Most candidates were able to find the correct product of prime factors. Some found the correct factors but listed them rather than writing them as a product and some included 1 in the product. A small number of candidates listed all factors of 54.

   (ii)   Many candidates did not connect this question with their prime factorisation in the previous part and those who did give the correct answer often achieved it by a trial and error approach. Considering the indices of the previous answer would have led directly to the answer of 4. It was common to see candidates attempt to find cube numbers greater than 54 and divide them to find a result.

(b)(i)   Many candidates identified that 3^3 is 27, but gave 3 as their answer having made no attempt to deal with the power for the square root. Correct answers in this part were rare, perhaps because candidates were not comfortable with fractional powers.

   (ii)   Candidates were more successful with the powers in this part and the correct answer was common. The most common incorrect answers were –6 and 5.

Answers: **(a)(i)** 2×3^3 **(ii)** *m* = 4 **(b)(i)** $k = \frac{3}{2}$ **(ii)** *k* = 6

Question 21

(a)(i)   Candidates who understood vector notation were often able to find $\overrightarrow{OC} = \mathbf{a} + \mathbf{b}$ however not all then understood that $\overrightarrow{OP}$ was one third of this expression. Some gave an answer without brackets such as $\frac{1}{3}\mathbf{a} + \mathbf{b}$ and others used $\frac{1}{2}$ in place of $\frac{1}{3}$.

   (ii)   Candidates who had given the correct answer in part **(a)(i)** usually gave a correct expression in this part, but it was sometimes left in the form $\frac{\mathbf{a} + \mathbf{b}}{3} - \mathbf{b}$ rather than in the required simplified form.

(b) There were two different approaches to this question, either a vector approach or an approach using the properties of the sides and angles of the triangles. Candidates using a vector approach often omitted the arrows from the vectors, in which case they were marked as sides rather than vectors.

Some did not take note of the direction of the vectors with the incorrect pair $\overrightarrow{OA} = \overrightarrow{BC}$ commonly seen.

Candidates who paired sides and angles usually gave the two required pairs of sides but often omitted the required pair of angles. Only a small proportion of candidates gave responses that gained full credit, though many gained 1 mark for some correct pairs stated.

Some candidates are clearly confused between similarity and congruence as they showed that the ratios of pairs of sides were equal. It was not uncommon for candidates to attempt to show that triangles *OAC* and *OBC* were congruent rather than the required triangles *OAQ* and *CBP*.

Answers: (**a**)(**i**) $\dfrac{\mathbf{a} + \mathbf{b}}{3}$ (**ii**) $\dfrac{\mathbf{a} - 2\mathbf{b}}{3}$ (**b**) shows triangles are congruent

Question 22

(a) Many candidates were able to find the height correctly. Some incorrect answers were due to arithmetic slips but an answer of 120 from 15×8 was not uncommon.

(b) Most candidates understood that they needed to find the sum of the areas of the faces to find the surface area of the container and the correct answer was seen reasonably often. It was common however for candidates to include a top for the container, leading to the answer 516 cm^2. Other candidates only added the area of one of each face or assumed that the areas of the vertical faces were all the same. Arithmetic errors were common in this part. A small number of candidates attempted to find either the volume or the perimeter of the container.

(c) Many candidates found the correct answer in this part by finding 60 per cent of their answer to part **(a)**. Some however used the longer approach of finding 60 per cent of the volume and then doing a second calculation to find the height of water which led to a more challenging calculation.

Answers: (**a**) 6 cm (**b**) 396 cm^2 (**c**) 3.6 cm

Question 23

(a) Many candidates were able to solve the equation correctly. Those that made errors usually made the correct first step of eliminating the fraction, but then made errors in either expanding the bracket or with signs when rearranging. Some candidates reached 16x = 12 but incorrectly rearranged to $x = \dfrac{16}{12}$.

(b) Most candidates understood that they needed to factorise the numerator and denominator in order to simplify the fraction. More successfully identified that the numerator was the difference of two squares than were able to correctly factorise the denominator. Incorrect factorisations of the denominator were usually in the form $(2x – 15)(x – 1)$. Only a small number of candidates did not factorise and attempted to cancel terms in the original numerator and denominator.

Answers: (**a**) $x = \dfrac{3}{4}$ (**b**) $\dfrac{2x + 3}{x - 5}$

Question 24

(a) Many candidates found the combination of probability and algebra in this part challenging. Some were able to reach a correct expression on the first branch but significantly more problems were seen with the probabilities on the second set of branches. Confusion with the order of subtraction was common, and expressions such as $\dfrac{3 - n}{n}$ were seen. Few candidates used expressions such as $1 - \dfrac{3}{n - 1}$ on the branches, which were acceptable.

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(b) Many candidates were able to select the correct product from the tree diagram and set up an equation for the probability. This was often successfully rearranged into the required equation with sufficient steps clearly shown, although some candidates were penalised for omitting = 0 in their final equation. A small number of candidates added the fractions rather than multiplying them and some candidates just solved the given quadratic equation in this part.

(c) Many candidates realised that they had to solve the given quadratic equation. This was often done successfully, usually by factorisation. Some candidates did not reject the negative solution which was invalid in this context of balls in a bag. Candidates who attempted to solve using the quadratic formula often made errors with signs or were unable to find the square root of 361.

Answers: **(a)** $\dfrac{n-3}{n}$, $\dfrac{n-3}{n-1}$ and $\dfrac{n-4}{n-1}$ correctly positioned on tree diagram **(b)** $\dfrac{3}{n} \times \dfrac{2}{n-1} = \dfrac{1}{15}$ correctly rearranged to $n^2 - n - 90 = 0$ **(c)** 10

MATHEMATICS D

Key messages

Candidates who performed well, in questions where numerical accuracy was required, remembered to work with sufficient figures throughout, so that their final answer was accurate to 3 significant figures.

General comments

The paper proved to be readily accessible to candidates across the whole ability range. In particular, the quality of the candidates' responses involving the accurate plotting of points and drawing of smooth curves, was of a high standard. Other areas in which candidates usually responded successfully were the questions involving matrices, Pythagoras's Theorem and the Cosine rule. Areas which candidates can improve on include the understanding of, and the drawing of, the tangent to a curve and calculating its gradient, the knowledge of and the application of circle theorems and the accurate construction and interpretation of loci.

Comments on specific questions

Section A

Question 1

(a) There were many fully correct responses given by candidates to this part question. Some however did not write their answer in its simplest form, but left it as 24 : 12 : 18.

(b) Many candidates found this part difficult and only a small number were completely successful here. Some candidates did use the ratio 2 : 2 : 1, but divided the initial quantity of 54 by this ratio, giving $\frac{2}{5} \times 54$, $\frac{2}{5} \times 54$ and $\frac{1}{5} \times 54$. Others did not use any strategy at all and various random numbers were given as their answers. Again, there was a small number of candidates who did realise that the highest values that could be used in the required ratio, were 10 : 10 : 5 **,** but they then did not go far enough and gave these values as the final answer.

Answers: **(a)** 4 : 2 : 3 **(b)** c = 14, v = 2, t = 13

Question 2

(a) Candidates who were successful here saw $36,720 as 102%. While there was a good number who arrived at the correct answer, many more were unsuccessful because they proceeded to calculate 2% of $36,720 instead and then either added or subtracted this value to or from $36,720.

(b) There were many excellent responses to this part, with full marks being awarded on many occasions. However, the common error made by a large number of other candidates, by misreading the question, was to subtract $12000 from $36,720 to get $24,720 and then candidates proceeded to find $\frac{2}{5}$ of this amount and 15% of it. A small number of candidates lost the accuracy mark by giving their answer to 2 significant figures instead of to 3 figures.

Answers: **(a)** 36000 **(b)** 12.3

Question 3

(a)   There were many candidates who successfully completed the table showing all the possible outcomes.

(b)(i) The answers to this part were usually correct.

(b)(ii) This part was not as well answered. Some did not show the correct fraction or gave the answer 0.66 instead of an answer to 3 significant figures. Others were confused with the correct denominator to use and answers of $\frac{6}{18}$ or $\frac{5}{18}$ were given.

(b)(iii) Again, this part was not as well answered as **(i)**. A common error here was to do the calculation $\frac{1}{3} \times \frac{1}{3}$ giving $\frac{1}{9}$ as the answer.

Answers: **(a)** GCB, HPC, HPB, HCB, RPC, RPB, RCB **(b)(i)** $\frac{3}{9}$ **(ii)** $\frac{6}{9}$ **(iii)** $\frac{2}{9}$

Question 4

(a)(i) Many fully correct answers were seen in this part on multiplying matrices.

(a)(ii) Whilst a large number of correct answers were given and most candidates calculated the value of the determinant correctly, a small percentage of candidates either did not manipulate the first and fourth elements correctly or did not multiply the second and third elements by –1.

(b)   Again, many candidates were completely successful in this part. Some candidates lost accuracy in subtracting the negative numbers in the matrices and it was common to see –2 instead of –1 as the fourth element in the answer matrix.

Answers: **(a)(i)** $\begin{pmatrix} 0 & 1 \\ 8 & 1 \end{pmatrix}$ **(ii)** $\frac{1}{4}\begin{pmatrix} -1 & 1 \\ -6 & 2 \end{pmatrix}$ **(b)** $\frac{1}{2}\begin{pmatrix} 4 & -3 \\ 14 & -2 \end{pmatrix}$

Question 5

(a)   The answers given for this part on corresponding *x* and *y* values for a function were nearly always correct.

(b)   Many candidates made good attempts at the drawing of a smooth cubic curve. There were a handful however, who incorrectly used a ruler to draw straight line segments. The plotting of the points was done accurately by the majority of candidates.

(c)   Only a small percentage of candidates gave the three required solutions. Many gave only two and some gave only one.

(d)   Correctly drawn tangents, with the correct gradient calculated, were seen only rarely. Many candidates did not make an attempt here. Some, who did draw the tangent, evaluated the gradient incorrectly and ended up with a negative value for their gradient instead of a positive value. Only on a few occasions, did candidates spoil their attempts at drawing the tangent, by leaving an 'air-gap' between the tangent and the curve. Similarly, one or two drew lines which were chords and not tangents.

(e)(i) A fair proportion of candidates did draw the required line successfully.

(e)(ii) Only the more able candidates knew that they had to equate the equations of the line and the cubic equation in order to be successful here. Weaker candidates just seemed to choose random values for *a* and *b*.

(e) (iii) Again, it was the more able candidates that gave acceptable solutions here. Although only one solution was asked for, some successfully gave both the values. Unfortunately one or two candidates gave the *y* value of –6, which is where the line and the curve intersect and not the *x* value as required.

Answers: **(a)** 17 **(b)** smooth curve **(c)** –1.7 to –1.4, –0.5 to –0.2, 1.7 to 2.0 **(d)** 3 to 5 **(e)(i)** correct line **(ii)** *a* = 7 , *b* = 4 **(iii)** –2.4 to –2.1 or –0.7 to –0.5

Question 6

(a) (i) This was a successful part for many candidates, with Pythagoras's Theorem being well known.

(a) (ii) Many candidates realised that trigonometry was required here and most were correct in arriving at an acceptable value for either angle *BDA* or angle *DAB***.** Successful candidates then went on to add the former to 90°, so obtaining the correct final answer. However, a good number of them forgot to add the 90°. Seen less, was 51.3° obtained for angle *DAB* which was sometimes also given as the final answer, or it was then added to 180° to give 231.3° for the final answer.

(b) (i) Very well answered with many fully correct answers seen.

(b) (ii) Quite well answered. Some, who measured the required angle incorrectly, usually did draw the length of *TF* to an acceptable degree of accuracy.

(b) (iii) Again, a good number of candidates were successful here. However, some measured the angle *TFS* and gave this as their answer, or they measured angle *TSF*. Some were closer and measured the angle *NSF* and used this for their answer, forgetting that a bearing is measured in a clockwise direction from North.

Answers: **(a)(i)** 14.4(2..) **(ii)** 128.6° to 129° **(b)(i)** 472 to 488 **(ii)** F correctly placed **(iii)** 242° to 248°

Question 7

(a) The vast majority of candidates had this part question on factorising correct.

(b) (i) In comparison, this part was answered correctly by only a small percentage of candidates. Some knew the factors to use but gave their answer as $(2x + 3)(2x + 3)$.

(b) (ii) It was common to see candidates using the quadratic formula here to solve the equation, especially if they did not realise the connection with this part and the previous part. Many were successful and obtained the correct solutions. Of the small number of candidates who did equate $(2x + 3)^2 = 49$ and went on to reach the correct solutions, others then stated incorrectly, that $2x + 3 = 49$ and went on to give *x* = 23 as their answer.

(c) Only reasonably well answered and usually by the more able candidates. The common error that was apparent on many of the candidates' answers, was not correctly expanding the brackets, e.g. $-(p – 3)$ became $-p – 3$ or $– 2(p – 3)$ became $-2p – 6$, leading in both cases to an incorrect final answer. A few candidates did not give their answer in its simplest form and left it as $\dfrac{(2p+10)}{8}$.

(d) This part was fairly well answered. There was a good number of candidates who gave a completely correct solution, but others were only partly correct. They sometimes reversed the inequality sign, or omitted it altogether or only gave the decimal equivalent answer incorrectly as *m* < –0.83 and not *m* < – 0.833 as required, correct to 3 significant figures.

Answers: **(a)** $3ab(4a – 5b^2)$ **(b)(i)** $(2x + 3)^2$ **(ii)** 2, –5 **(c)** $\dfrac{p+5}{4}$ **(d)** $m < -\dfrac{5}{6}$

Question 8

(a)   This part on continuing the pattern was nearly always correct.

(b)   Again, many candidates gave completely correct values in the table.

(c)   This was not so well answered as the two previous parts, but still a good proportion of candidates were able to work out the correct expression.

(d)   Those candidates that were successful in the previous part nearly always obtained the correct answer here too.

(e)   This was not answered well by many candidates. This was because candidates, on the whole, did not start by finding the correct expressions for the number of tiles in both the outer row or column. Only a handful of the most able of candidates did this. The most common approach was to add the number of grey tiles, previously found in **(c)**, to the number of white tiles '$4n^2 + 6n$', thus obtaining the required total.

(f)   Candidates were mostly successful in this part too, especially those that had used the required expression in their approach to answering the previous part.

(g)   Weaker candidates did not equate the total number of tiles to being equal to 8 times the number of grey tiles. Those candidates that did so, were usually then able to form the resultant quadratic equation and successfully factorise it, to obtain the correct answer of $p = 7$. Only a few candidates spoiled their answer by giving both the positive and negative solutions, not realising that it was necessary to reject the latter value.

Answers: **(a)** correct diagram **(b)** $\begin{matrix} 22 & 26 \\ 88 & 130 \end{matrix}$ **(c)** $4n + 6$ **(d)** 26 **(e)** $(2n + 3)(2n + 2)$ **(f)** $4n^2 + 6n$ **(g)** 7

Question 9

(a)   This question on polygons was not answered well by many candidates. Many reached 1260° and thought that this was the final answer, not realising that it was necessary to divide this value by 9, to find the interior angle.

(b) (i)   The need to use the Cosine rule here was recognised by the vast majority of candidates and there were many fully correct answers seen. One or two candidates misquoted the rule, either through putting a '+' where the '–' should be or omitting the '2' from the product '$2ab\cos\theta$'. The other reason that some candidates did not succeed completely was through not showing that the answer was 21.89… in their working, but having reached the correct square root stage, they then gave as their answer 21.9 without showing the required intermediate step.

(b) (ii)   The use of the Sine rule was also well recognised by candidates and there were many fully correct answers given here as well. A small number of candidates lost the accuracy in their calculation, by not working with sufficient figures e.g. $\sin^{-1}(0.32)$ giving as the answer 18.66.

(b) (iii)   Though a good number of candidates knew to use the rule $0.5ab\sin\theta$ here and equated this to 109 cm², in order to arrive at the value of angle *DEB* as 84.5°, there were many who gave this value as their final answer. Only the more able candidates realised that the obtuse angle required meant that they had to subtract 180° – 84.5°. A few others incorrectly subtracted 360° – 84.5° instead.

Answers: **(a)** 140° **(b)(i)** 21.89…. **(ii)** 18.8° to 19° **(iii)** 95.47° to 95.5°

Question 10

(a) (i)   Many candidates gave the correct value of 60°, but there was only a small percentage of candidates who were able to give the fully correct reason. Some were close to being correct, but mistakenly stated that 'the angle at the circumference is twice the angle at the centre'. Others used incorrect terminology such as, 'angle at the middle' or 'angle on the edge'.

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(a) (ii) Only the more able candidates realised that it was necessary to use the fact that the sum of the angles of the quadrilateral *DAOB* was 360° and in so doing, they were able to correctly obtain the required answer. There were a good number of candidates who did realise that triangle *AOB* was an isosceles triangle and were thus able to obtain some credit, for giving either angle *OAB* or *OBA* as being 30°. A smaller number did gain credit for knowing that the reflex angle *AOB* was 240°.

(a) (iii) That candidates had to know that the sum of opposite angles of a cyclic quadrilateral is 180°, was a fact that appeared to be not well known. Correct answers were not often seen here.

(b) (i) The better able candidates were able to set up the required equation equating the area of the sector OAB to the area of the circle of radius '*r*'. Many of them then successfully were able to expand the brackets and reach the required quadratic equation equated to 0.There were only a handful of candidates who were penalised for not showing '= 0', as was required. Weaker candidates thought that they had to factorise the given quadratic equation, or else they showed that they could expand $(r + 4)^2$, but did not know the full strategy that was required.

(b) (ii) There was generally more success here, as candidates recognised that they had to use the quadratic formula to calculate '*r*' and this method was shown to be well known. Most successful candidates knew to choose the positive solution only in this context, but one or two incorrectly gave both the positive and negative solutions.

Answers: **(a)(i)** 60° **(ii)** 70° **(iii)** 110° **(b)(ii)** 5.46 to 5.47

Question 11

(a) There were a good many fully correct answers seen here, with candidates showing that this method is generally well understood. However, a common error, seen on quite a number of occasions, was the use of the class width, and not the mid-point, being multiplied by the frequency. Or, at other times, candidates multiplied either the lower end value or the upper end value of each class by the frequency.

(b) Generally completed successfully by nearly all candidates.

(c) Most candidates drew good or very good cumulative frequency curves.

(d) (i) The median value was shown to be well understood, as there were many correct answers given.

(d) (ii) Also generally well understood by many candidates. Although, a misunderstanding seen on several occasions, was for candidates to realise that they needed to use the 25th and 75th percentile values, that is 20 and 60 respectively, but then they did 60 – 20 to get 40 and then read off from 40 on the cumulative frequency axis to the curve, to get the value on the Amount axis.

(e) Only the most able of candidates knew what was required here and there were many instances where candidates made no response. Some did manage to work out the number of people who spent more than $85, approximately 28, from correctly reading the graph, but then became confused with what was required next.

Answers: **(a)** 75 **(b)** 25, 46, 64, 73, 78 **(d)(i)** 74 to 76 **(ii)** 36 to 44 **(e)** 54 to 62

Question 12

(a) (i) Whilst there were many accurate constructions seen, a fair number of candidates could not have read the question correctly, as there were a number of instances where candidates constructed point D to the right of point B.

(a) (ii) A large number of candidates measured angle DAB correctly. However, on quite a number of occasions candidates mistakenly measured angle DAC instead. Of those candidates who constructed point D to the right of point B, some managed to redeem themselves by correctly measuring their angle DAB.

(iii)(a) This part was usually correct, but some candidates drew lines from B to AC that were obviously not at right angles to one another, so could not be the shortest distance between B and AC.

(b) Those candidates who were correct in the previous part usually gave a correct answer here too, using $0.5 \times AC \times$ their **(a)**. However, some candidates measured the angles of triangle ABC and the lengths of the sides and correctly used $0.5\ ab\sin\theta$, with the appropriate measurements used, to obtain an acceptable answer. There were also a few candidates who used $0.5 \times BC \times AB$, incorrectly assuming that angle *ABC* was 90°.

(b)(i) This part was not answered well by a large number of candidates. It showed that many candidates were not aware that opposite pairs of angles in a cyclic quadrilateral add up to 180°. Many responses made reference to either the sides being unequal, or the diagonals are unequal.

(ii)(a) Generally constructions were poor. Weaker candidates frequently made no attempt, while some were only able to attempt the arc for the locus from *R*, even then, some only gave incomplete arcs. Some did manage the arc and the angle bisector of angle *PQR*. Others knew how to construct the perpendicular bisector of *PR*. Fully correct, complete constructions, with the correct region shaded, were only infrequently seen.

(b) Following on from **(a)**, it was usually the more able candidate who gave an accurate, acceptable answer.

Answers: **(a)(ii)** 45º - 48º **(a)(iii)(a)** 2.9 – 3.1 **(a)(iii)(b)** 19.1 – 20.8 **(b)(ii)(b)** 7.9 – 8.3

MATHEMATICS D

Paper 4024/22
Paper 22

Key Messages

Final accuracy marks were often lost where answers were not rounded correctly to 3 significant figures, with the most common error being truncated answers, or where premature rounding had occurred.
Candidates should be discouraged from writing their answer in pencil and overwriting it in pen. These answers become very difficult to read when they are scanned for marking and it is not always clear what digit is meant by some of their answers. This practice is also not good use of the candidates' time.

General Comments

Most candidates appeared to complete the examination without a shortage of time. The candidates seem to have been well prepared for this exam. Overall there were fewer candidates being entered at a level beyond their ability. In general, **Question 9** appeared to be the question that many chose to omit, but many who answered this question scored high marks on it. **Question 11** appeared to be one that the candidates were confident in answering, as it was routine in nature until **(b)(iii)**.

Comments on Specific Questions

Section A

Question 1

(a)   The most able candidates scored full marks here. Others struggled to gather all the information. It was common to see candidates forget to multiply by 14 when calculating the total cost of the accommodation. Some candidates calculated the cost for the flight and accommodation but did not know how to deal with the insurance, some not using the cheapest one and some thinking there was a need to calculate the cost of the insurance per day and then calculate 14 days. There were some candidates who thought they needed to use lower bounds for this question.

(b)   In general, candidates answered this part with more accuracy. Some worked out the cost for both companies but then did not answer the question as they did not calculate the difference. The calculation for Value Cars proved more difficult. Some candidates obtained answers that did not make sense and candidates could have realised that their answer was wrong when the cost of hiring a car for 14 days was more than, for example, $300 000.

Answers: **(a)** 9370 **(b)** Bonus cars by $67

Question 2

(a)   Most candidates gave a correct response with a few choosing to write their answer in standard form. A significant number of candidates misunderstood the wording of the question and calculated how many times larger the population of Pakistan was compared to the population of South Korea. Other common wrong responses were 138 404 and 183 192 500.

(b)   Almost all candidates gave Thailand as their answer with only a few opting for South Korea.

(c)   Many candidates gave correct answers here, with the majority of these giving the answer correct to 3 significant figures. There were those who chose to give their answer correct to either 2 significant figures or 1 significant figure. Occasionally the answer had the power as –7.

(d) This part was found more challenging. The correct answer was nevertheless seen on many scripts but accuracy was as usual the governing factor which determined whether full marks would be earned. A common wrong answer was 1.63 after a correct method. The most common wrong approach was to divide by 188 169 000 leading to an answer of 1.613.

(e) The large numbers, the accuracy requirement and the difficult concept of reverse percentages made this part a particularly daunting assignment for many candidates. There were an encouraging number of fully correct solutions but the wrong method which was commonly seen involved finding 1.68% of 15 677 000 and subtracting it from 15 677 000 to give 15 413 626.4. There were also cases where 15 677 000 was multiplied by 1.0168 or 0.0168 or divided by 0.0168. Candidates who aligned 15 677 000 with 101.68% generally went on to 15 417 977.97 and many successfully rounded this to 3 significant figures.

Answers: **(a)** 138 404 000 **(b)** Thailand **(c)** 4.9512×10^7 **(d)** 1.64 **(e)** 15 400 000

Question 3

(a) A correct table was completed by many candidates. There were a few who completed the 4th row correctly and then had 5, 10, 15, 20 for the 5th row, others had the 4th row as 6, 9, 12, 15 and the 5th row as 8, 16, 32, 64, while some had the 4th row as 6, 12, 24, 48 and the 5th row as 8, 16, 32, 64.

(b) This was well done with most candidates scoring the mark. Occasionally answers of $\dfrac{4}{16}$ or $\dfrac{6}{16}$ were seen.

(c) Less wrong answers were seen from candidates in this part. The majority obtained the correct fraction and simplified it correctly.

(d) Although most candidates knew what a square number was, many failed to find all six that appeared in the table, often missing the 1 or not including all the 4's. Many gave the number of factors of 6 as 6, usually failing to write down that 1 is a factor of 6. Some confused factors with multiples. Many candidates dealt with either the squares or factors of 6 correctly, usually stating the probability, but often not dealing correctly with both.

Answers: **(a)** 3 6 9 12 / 6 8 / 4 8 12 16 **(b)** $\dfrac{5}{16}$ **(c)** $\dfrac{3}{4}$ **(d)** No squares $\dfrac{5}{16}$ and factors $\dfrac{7}{16}$

Question 4

(a) Many correct answers were seen, however some candidates had problems manipulating the negative numbers. A few candidates worked out either 3B – 2A or 3A – 2B.

(b) There were many correct answers here with candidates giving the correct elements in a matrix of the correct order. Some candidates had a matrix of the wrong order, giving a 1×2 matrix for their answer. Problems were seen with the manipulating of negative numbers so it was not unusual to see only one element of the matrix correct. Several candidates gave their answer as a 2×2 matrix, often because they did not add each pair of elements together.

(c) Many candidates knew how to find the inverse of a matrix. Errors were occasionally seen with the determinant due to poor manipulation of the negative numbers leading to a determinant of 14, –14 or –2. Candidates who had the correct inverse had varied success with completing the question. Some gave their answer as the inverse. The most common error, from the candidates who attempted to add A to its inverse, was to add the adjoint to A and then divide the result by the determinant.

Answers: **(a)** $\begin{pmatrix} 1 & 0 \\ 8 & 8 \end{pmatrix}$ **(b)** $\begin{pmatrix} -7 \\ 5 \end{pmatrix}$ **(c)** $\begin{pmatrix} 2 & 1 \\ -2 & -\frac{1}{2} \end{pmatrix}$

Question 5

(a) Candidates often had problems cancelling to the simplest form. Common wrong answers were $\dfrac{9x}{10}$ (often after a correct answer), $\dfrac{9x}{10x}$, or those with insufficient simplification such as $\dfrac{5x+4x}{10x^2}$ or $\dfrac{9x}{10x^2}$

(b) This was usually done well. The wrong answer of $7x - 11y + 5$ was often seen, due to the minus sign not being applied to all elements within the second set of brackets. Another common wrong answer was $7x - 5y + 5$.

(c) Most candidates knew to use the quadratic formula and gained some credit but many did not get fully correct answers. Problems arose mainly with the substitution of $b = -1$; others used $b = -1$ but did not square or multiply correctly in the first term of the numerator. Another common error was not to extend the division line under the first term. Quite a few candidates had an incorrect denominator of 2 and not 6. Many candidates had problems rounding otherwise correct answers, often truncating instead. Some attempted the question by trying to factorise the quadratic.

(d)(i) Although a lot of candidates got this correct, there were still a significant minority who produced lines with a positive gradient. The line $x + y = 5$ was commonly seen as was $y = 2.5$.

(ii) A high proportion of candidates who knew how to draw the line correctly identified the region. Some candidates did not make it clear which region they were labelling R.

Answers: **(a)** $\dfrac{9}{10x}$ **(b)** $7x - 5y + 3$ **(c)** -1.14, 1.47 **(d)(i)** $x + 2y = 5$ drawn **(ii)** correct region identified

Question 6

(a) This question was well answered. A large majority of candidates recognised they needed to use sin 38 which usually led to the correct answer 7.39. There were errors seen when candidates did not round their answer correctly to 3 significant figures; truncating to 7.38 was the most common wrong answer while others rounded to 2 significant figures. Some candidates mistakenly used cosine and a small number used tangent.

(b) This part of the question caused many problems with only a small minority scoring full marks. This was mainly caused by candidates not placing the point S correctly to the right of R on the same horizontal baseline. The question required PRS to be a right angle which seemed to suggest for some that $PQSR$ was a square. Many placed S somewhere along the top line. The incorrect diagram usually meant incorrect trigonometry was applied. For some this resulted in an angle of 128 instead of 52 being used in the sine rule. The whole concept of two possible values for angle QSR, especially the obtuse one, seemed difficult to grasp. A vast proportion who correctly got 71 did not understand that both answers needed to add up to 180. Some candidates produced a second calculation, which may have produced another answer close to 71 with a different degree of accuracy or stated the other answer as 90. Some candidates chose to calculate PR and used this with 10 and cosine to calculate the angle, however this frequently led to an inaccurate final answer.

Answers: **(a)** 7.39 **(b)** 71.0, 109.0

Question 7

(a) Many correct answers were seen, with some exceptionally well laid out diagrams. However, a significant minority struggled to produce a pattern that was in reasonable alignment, with the worst consisting of randomly placed circles and crosses that bore no resemblance to the previous patterns in the sequence.

(b) Almost all candidates completed the table correctly, even in cases where pattern 5 was not drawn or was drawn incorrectly.

(c) Many recognised that the bottom row of the table contained square numbers. Others defined a quadratic expression for the nth term and proceeded to evaluate the coefficients. Some simply used the information given in later parts of the question.

(d) This part was almost always correct.

(e) A wide variety of methods were seen with the highest achieving candidates obtaining an expression without difficulty. The most common methods started from $n^2-(\frac{1}{2}n^2+\frac{1}{2}n)$, an^2+bn+c or $a+(n-1)d+\frac{1}{2}(n-1)(n-2)c$, although some using these methods made mistakes on the way. Many candidates seemed unclear what was required and answers that verified the given expression for values of n were often seen.

(f) This part proved to be a good discriminator. Some excellent solutions using efficient algebraic methods were seen but this was not true of most responses. Although the word 'crosses' was highlighted in bold type, a considerable number of candidates used the expression for circles. A significant proportion of those starting correctly arrived at $m^2 = 9m$ and did not know how to proceed, most opting to take the square root of both sides. This usually led to the incorrect answer of $m = 3$.

Answers: **(a)** correct pattern **(b)** $\begin{matrix} 15 & 21 \\ 10 & 15 \end{matrix}$ **(c)** n^2 **(d)** 465 **(e)** $\frac{1}{2}n^2-\frac{1}{2}n$ algebraically shown **(f)** 9

Section B

Question 8

(a) Some candidates did not understand that the way to solve this was to consider the equivalent line from *B* parallel to *DC*. Those who did usually went on to use Pythagoras' theorem effectively. The most common error involved rounding, with many giving the truncated answer of 14.9. Candidates need to appreciate that writing an answer to 3 significant figures does not mean writing down the first 3 digits. Some candidates recognised that trigonometry could be used and gave the correct method for finding an angle and used this to find *DC*. Candidates who used this method often gave rounded values for intermediate steps and then rarely arrived at an accurate final answer.

(b) Candidates generally recognised that the cosine rule was required and many fully correct solutions were seen. Occasionally the sides were substituted in incorrectly resulting in one of the other angles being calculated. The most common problem with the candidates who had substituted correctly was dealing with the negative value, a significant number of candidates either lost the minus sign or simply ignored the fact that their cosine was negative, leading to an incorrect answer of 82.5°. The weakest candidates who started with the correct implicit expression for the angle at *A* often made errors in their manipulation. These candidates often reduced $202 - 198\cos A$ to $4\cos A$.

(c) Although many correct methods were seen, a significant number of candidates were using inaccurate values from **(a)** and / or **(b)**. Most showed a clear method with more opting to find the area of *BCDE* by splitting it into a rectangle and a right-angled triangle. The most common error in finding the area of *BCDE* was to have the perpendicular distance between *DE* and *BC* as 15.1. Many found the area of *ABE* by using $\frac{1}{2}ab\sin C$, however some assumed this was a right-angled triangle and so $\frac{1}{2} \times 9 \times 11$ was often seen.

(d)   This was a question that discriminated well, with fully correct answers to this part being rare. Most realised that a conversion of units was required, although many were unable to do this correctly. However, only a tiny minority understood the need to find the square of the given linear factor to apply it to an area.

Answers: **(a)** 15.0 **(b)** 97.5 **(c)** 124 **(d)** 496

Question 9

(a)   Those candidates who initially formed the product of (x + 2) and (10 – x) almost always went on and successfully reached the quadratic required. A small number failed to include the y, but the most common mistake which prevented candidates from scoring any marks was to write x – 10 instead of 10 – x.

(b)   The quality of the graphs was of a high standard overall, with the majority drawing a single freehand line through their points. Despite a table not having been given, most candidates found y values at each of the integer values of x from 2 to 8 inclusive. The points which were most commonly plotted wrongly were those at (2, 32), (6, 32) and (7, 27), the scale on the y-axis being incorrectly read. Some of the very weakest candidates did not seem to understand what was required; these usually attempted to draw a graph using only the points they had been given.

(c)   Many candidates who realised the need to draw the line y = x scored full marks, although a significant number read the intersection value for x incorrectly, answers of 9 being the most common. It was not clear whether these candidates simply made errors in reading the value from the graph, or whether they were under the impression that Manuel's number had to be an integer. An answer of 9 was also seen with the line y = 9 drawn. Others gave a correct answer but had not drawn a line, with the answer space being blank or random numbers inserted.

(d)   The expression 5x + 2 was commonly seen or there was clear evidence of its use on most scripts where this question was attempted. Progress from this point was varied, methods using trial and improvement being quite commonly utilised, and quite often such an approach led to x = 6. The more able candidates knew to equate the two expressions, usually making some progress with rearranging it to a 3-term quadratic. Unfortunately, a significant number of candidates, faced with an equation such as x^2 – 3x = 18, seemed unclear how to proceed. Some attempted to use the quadratic formula without rearranging the equation and others who did rearrange often made sign errors resulting in the wrong solutions.

Answers: **(a)** (x + 2)(10 – x) leading to answer **(b)** correct curve **(c)** 9.2 with y = x drawn **(d)** –3, 6

Question 10

(a) (i)   This part of the question was completed well with many candidates scoring full marks for placing points B and C the correct distance and bearing from A. Errors made were more commonly in measuring the bearing from A than in measuring the distances from A. Also noticeable was the larger number of mistakes in the positioning of C than B, probably due to the angle being more than 180°. Many of the constructions reflected neatness, accuracy and concern for presentation.

(ii)   The positioning of D was generally well done, candidates appreciating that D's shortest distance from A was when AD was perpendicular to BC. Common mistakes were to draw AD horizontal, bisect angle CAB or bisect BC. There were occasions when candidates drew a line from A perpendicular to BC however they did not draw the point D on BC.

(iii)   It was almost always the case that if candidates were successful in **(a)(i)** and **(a)(ii)** then they measured the distance accurately.

(iv)   Most of the candidates who had D correctly placed gave the ratio in the required form. There were those who measured accurately but were not able to do the necessary division to obtain the value of n. Some of the candidates who had a point D placed on BC gave a ratio in the correct form, appropriate to their positioning of D.

(v) This part proved too difficult for all but the best candidates and very few scored the mark. Many candidates squared their ratio of lengths found in **(a)(iv)** and many others did not have an answer involving *w*. Candidates did not appear to understand the relation between this part and **(a)(iv)**. Very few appreciated that, because the two triangles had the same perpendicular height, their areas were proportional to their bases. Only a small number presented their answer as the difference between *w* and their area of triangle *ABC*.

(b) Candidates scored well in this part, constructions again being neatly presented for the most part. Understanding the conditions which define the region and linking this to the three different loci constructions was the crux of the question. It was encouraging to see how well many candidates appreciated this and then went on to identify the region successfully. Precision was also very good, the perpendicular bisector being the loci which, although generally accurate, was less so than the other two.

Answers: **(a)(i)** *B* and *C* drawn **(ii)** *D* drawn **(iii)** 2.9 **(iv)** 1.3 **(v)** 0.769*w* or 18.85 – *w* **(b)** correct region

Question 11

(a) (i) The modal class was well known; just a few gave the frequency 16 with their answer or as their answer. Some gave $60 \leqslant t < 65$ as their answer which was mid-placed in the table, possibly confusing modal class with the median.

(ii) This part of the question was well answered with many scoring full marks for correctly calculating the mean from a grouped frequency table. The usual errors were seen by the weaker candidates in using incorrect mid-points; multiplying by the upper end of each interval, the interval width of 5 or wrong/inconsistent values. A small number of candidates made an arithmetic slip in their calculation but showed the method they were using.

(iii) Most candidates found the correct probability. Errors were seen in the denominator e.g. 7 + 16 = 21 or spoiling of the numerator by adding a third number e.g. 7 + 16 + 15. A few tried to multiply probabilities e.g. $\dfrac{23}{50} \times \dfrac{22}{49}$ or $\dfrac{7}{50} \times \dfrac{16}{50}$

(b) (i) Candidates almost always read the correct median, 34, from the cumulative frequency curve. A few gave the answer 35, possibly stating the midpoint of the Time axis.

(ii) A high proportion of candidates calculated the correct interquartile range from the cumulative frequency graph and most made a good attempt at it. Some lost accuracy on one or both readings. A few candidates subtracted the cumulative frequency values UQ 60 – LQ 20 = 40 and either gave 40 as their answer or read from the graph at 40.

(iii) Candidates found this question difficult and it proved to be a worthy discriminator. A minority of candidates scored full marks. Several chose not to answer this part and many plotted only 1 or 2 points, usually the first and last. Candidates did not understand how to use the information given regarding the lower quartile and interquartile range to work out the middle two points. It seemed that some candidates appreciated what the general shape should look like and filled in the central part by eye. Some made the mistake of assuming the curve started at (25, 0).

Answers: **(a)(i)** $55 \leqslant t < 60$ **(ii)** 60.8 **(iii)** $\dfrac{23}{50}$ **(b)(i)** 34 **(ii)** 4.5 **(iii)** cumulative frequency curve drawn

Question 12

(a) This part was often well done, although some candidates stopped at finding the arc length and did not proceed to find the perimeter of the sector. Some candidates found the lengths of the straight lines *AC* and *CB* rather than the arcs, whilst others found the area of the sector. Again, there was inaccuracy produced by truncating the arc length to 12.5.

(b)(i) This part was accessible for most candidates. The use of pi as 22 / 7 caused inaccuracies in the answer given; candidates must be encouraged to follow the instructions on the cover sheet instructing them to use the calculator value for pi or use 3.142.

(ii)   Many correct answers were seen by candidates, with candidates realising the need to work out the area of two triangles using $\frac{1}{2}r^2\sin 36$. This was usually subtracted correctly from **(b)(i)**. Mistakes were seen by candidates who worked out the area of a triangle of angle 72 and subtracted this from the area of the sector. Others thought the two triangles were right-angled triangles and so calculated *AC* or *BC* using this assumption before calculating the area of the triangle.

(c)   Due to an issue with this question, a discussion took place at the examiners' meeting that took place before marking began, and examiners considered the impact on candidates. Changes to the marking approach for this question were agreed to ensure that no candidates were disadvantaged by the error.

Answers: **(a)** 32.6 **(b)(i)** 62.8 **(ii)** 4.05

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